FunktionWin

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Dietmar G. Schrausser orcid.org/0000-0002-4924-8280
Karl-Franzens University, Graz, Austria

Graphical MS Windows user interface for ConsoleApp_DistributionFunctions (Schrausser, 2024b).

The following functions were realized, German notation:

1. Wahrscheinlichkeits-Verteilung (probability distribution)

- Binomial-Funktion $f(X = k|n)$.
- Poisson-Funktion $f(X = k|n, p)$.
- Geometrische-Funktion $f(X = r|p)$.
- Hypergeometrische-Funktion $f(X = k|n, K, N)$.
- Exakt *binomialer* 2-Felder Test $f(X = b|b, c)$.
- Exakt *hypergeometrischer* 4-Felder Test $f(X = a|a, b, c, d)$, *Fisher Exact* (Fisher, 1922, 1954; s. Agresti, 1992).

The fundamental *binomial distribution* was derived by Bernoulli (1713), s. Schneider (2005a) and above all de Moivre (1711, 1718) with the discovery of the first instance of central limit theorem, to *approximate* the *binomial* distribution with the *normal* distribution, further developed by Gauss (1809, 1823), see Hahn (1970), Hald (1990) or Schneider (2005b).

2. Theta-Verteilung θ (characteristic value or θ distribution)

- z-Dichte Funktion $f(x = z)$.
- z-Funktion $F(x = z)$.
- t-Funktion $F(x = t)$.
- χ^2- Funktion $F(x = \chi^2)$.
- F-Funktion $F(x = F)$.
- Effekt-Stärke ϵ, Cohen (1977).

The *t-distribution* was first derived by Lüroth (1876), later in a more general form defined as *Pearson Type IV* (Pearson, 1895), commonly known as *Student's t-distribution*, from William Sealy Gosset (1908).

Helmert (1876) first described the *χ^2-distribution*, independently rediscovered by Pearson (1900), c.f. also Elderton (1902), Pearson (1914) or Plackett (1983), for the *F-distribution* by Fisher (1924) see Snedecor (1934) and Scheffé (1959).

Statistical *power* $1 - \beta$ and *effect size* ϵ (Cohen, 1977, 1992) layed foundations for statistical meta-analysis and methods of estimation statistics, see e.g. Borenstein et al. (2001) for related software applications.

3. Transformationen (transformation functions)

- Fisher Z Funktion $F(x = r)$, Fisher (1915).
- Gamma $F(x) = \Gamma$.

Gamma Γ, to solve the problem of *extending* the *factorial* to *non-integer* arguments, was first considered in a letter from Bernoulli to Goldbach (Bernoulli, 1729), introduced later by Euler (1738) - of fundamental definitional importance for the formulation of *approximate probability distribution* functions such as χ^2, t or F (c.f. Meyberg & Vachenauer, 2001; Cuyt et al., 2008; Beals & Wong, 2020; Little et al., 2022).

See further e.g. Bortz (1984), Bortz and Weber (2005), Bortz and Schuster (2010), Döring (2023), Pascucci (2024a, b) and Schrausser (2024a).

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